Yeshiva University
KATZ SCHOOL

Mathematical Sciences, PhD

Except as noted, all courses are 3 credits. With the exception of Topics courses and Problem seminars, the first semester of any two-semester course will comprise a survey of the important topics in the course. The second semester, offered only upon sufficient student demand, will consist of a deeper treatment of topics covered in the first semester.

Elementary Theory of Numbers
MAT 5115
Divisibility, congruence, quadratic reciprocity, elementary results in quadratic forms, diophantine equations, and rational approximation to irrationals. Applications to cryptology and data security.

Linear Algebra
MAT 5117
Vector spaces, basis, dimension, direct sums, factor spaces; linear transformations, functionals, dual spaces, matrices, determinants; systems of linear equations; diagonalization, normal and canonical forms, elementary divisors; bilinear and quadratic forms; inner products, euclidean and unitary space, orthogonal and symmetric matrices; tensors and exterior algebra. Applications to Markov chains and linear regression.

Introduction to Analysis
MAT 5118
A survey of analytic methods which are of practical significance for applications, as well as the mathematical foundations, contexts, and limitations of those methods.

Functions of a Complex Variable
MAT 5127
Integration and differentiation in the complex domain. Cauchy's Theorem, Cauchy Integral Formula, Laurent expansion, residues. Elements of conformal mapping, special functions, series and product representations.

Problem Seminar
MAT 5200, 5201
Students are trained in applying their knowledge in various areas to the solution of specific problems arising in industrial and technological applications of mathematics: operations management, risk theory, shock wave theory, atomic force microscopy, materials science.

Ordinary Differential Equations
MAT 5209
Differential and integral equations in the real domain; existence and stability theory, Sturm-Liouville problem for linear equations, techniques of solution for special classes. Differential and integral equations in the complex domain; equations of Fuchsian type and special functions; transform methods. Transition to chaos.
Partial Differential Equations I, II, III
MAT 5210, 5211, 5215
Introduction to the theory of partial differential equations of second order. Problem of Cauchy, boundary value problems of potential theory, variational principles. Equations of mixed type. Applications to finance, geometry, plasma, and gas dynamics.

Functional Analysis I, II
MAT 5230, 5231
Banach and Hilbert spaces, linear functionals, Hahn-Banach theorem, dual spaces, linear operators, closed graph theorem, Riesz theory for compact operators, spectral theory.

Algebra
MAT 5253
Sets, Boolean algebra, cardinal numbers. Groups, rings and ideals, integral domains, fields, algebraic number fields, Galois theory, combinatorics. Diverse applications to computer science.

Topics in Probability Theory I, II
MAT 5256, 5257
Probability and risk measures. Applications to financial decision-making, operations management, and the theory of gambling. Random walks, Brownian motion, fractional Brownian motion , white noise processes, Markov chains, Markov processes, time series, convexity methods. Nonperiodic cycles and processes with memory.

Topics in Geometry
MAT 5258
Synthetic geometry. Projective spaces. Elements of algebraic geometry. Applications are chosen from computer graphics, finite-element analysis, geometrical optics, and the theory of caustics.

Differential and Riemannian Geometry I, II
MAT 5259, 5260
Classical differential geometry of curves and surfaces in space. Intrinsic geometry on a surface. Tensor calculus with applications to geometry in n dimensions. Elements of geometric analysis (harmonic maps). Applications to special and general relativity.

Topics in Modern Differential Geometry

MAT 5261
Definition and elementary properties of Lie groups and Lie algebras; vector bundles and connections. Morse theory. Elements of Hodge theory. Applications to high-energy physics and gauge-field theory.

Topology I, II
MAT 5262, 5263
A rigorous introductory treatment of point-set topology, differential topology, homotopy and homology.

Functions of a Real Variable
MAT 5265
Fundamentals of real analysis and applications. Lebesgue measure and integral. Integrals on sigma algebras. Probability measures. Introduction to Hilbert spaces and LP-spaces. Applications to Fourier series and to Fourier and more general transforms.

Mathematical Statistics
MAT 5266
Development of statistical models as corollaries of theorems in probability, and a rigorous presentation of topics related to the practice of statistics and data analysis.

Convex Optimization
MAT 5267
Convex analysis in finite dimensions; linear programming; convex optimization with constraints; vector (multi-criteria) optimization problems from theoretical and computational perspectives. Applications to finance and economics, including convex risk measures, portfolio optimization and utility maximization problems.

Data Science: Fundamentals and Applications
MAT 5270
Statistical and computational fundamentals that form the basis for contemporary data science applications in biomedical science, finance and other cognate 'big data' disciplines are introduced. Core components include data exploration, data modeling, the use of data mining technologies and application examples. Course material will be complemented by hands-on programming experience, using the iPython programming environment, to allow the class to gain a hands-on experience of data science analytics.

Applied Data Science: Contexts and Methodologies
MAT 5272
Examination of exemplar data science publications from the domains of biomedical science, quantitative finance, geoscience and the astronomical sciences.

Topics and Problems in Analysis
MAT 5301, 5302

Techniques of problem-solving and estimation, and related concepts in real and complex analysis. An introduction to working analysis in distinction to theoretical analysis. Financial and engineering applications are emphasized.

Topics in Partial Differential Equations I, II
MAT 5310, 5311
General theory of linear partial differential equations. Semilinear, quasilinear, and fully nonlinear equations. Variational theory. Cauchy and boundary value problems, Estimates and regularity of solutions. Topics will be chosen from contemporary application to geometry, finance, continuum mechanics, plasma physics, and engineering.

Mathematical Logic and Computability Theory
MAT 5312
Boolean logics, truth functions, quantification theory, Turing machines. Horn algebras, lattices, quasivarities. Applications to computer science and, in particular, artificial intelligence.

Readings in Mathematical Logic
MAT 5315
Topics to be arranged, depending on the interests and backgrounds of the students. Given only by arrangement with the instructor.

Readings in Linear/Modern Algebra
MAT 5317
Topics to be arranged, depending on the interests and backgrounds of the students. Given only by arrangement with the instructor.

Complex Systems
MAT 5320
Nonlinear and fractal time series; computational methods; applications include econophysics, fractal statistics, and neural physics.

Topics in Functional Analysis
MAT 5330, 5331
Hilbert and Banach spaces, operator theory. Applications.

Fourier Analysis
MAT 5365
Fourier integrals. Applications to signal processing, imaging science, time series analysis, and the theory of waves.

Graduate Seminar
MAT 5931, 5932

Faculty-supervised reading/research on a topic in contemporary mathematics.

Readings in Analysis
MAT 6401, 6402
Reading course for doctoral students. Variable credit.

Readings in Algebra/Geometry
MAT 6431, 6432
Reading course for doctoral students. Variable credit.

Readings in Mathematics
MAT 6451, 6452
Reading course for doctoral students. Variable credit.

Thesis Preparation
MAT 7705
Prerequisite: a grade of Pass on the Advanced Qualifying Examination. Variable credit.